Comments on: "Weinberg's Nonlinear Quantum Mechanics and Einstein-Podolsky-Rosen paradox", by Joseph Polchinski.

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Abstract

Contrary to the opinion of J. Polchinski [Phys.Rev.Lett. **66**, 397-400 (1991)], the phenomenon of superluminal messages in nonlinear versions of quantum mechanics is not a specific difficulty in a class of theories formulated by S. Weinberg [Ann.Phys. (N.Y.), **194**, 336-386 (1989)]. It appears in all schemes which try to enlarge the orthodox class of observables, while conserving the traditional structure of the pure and mixed states.

In the last decades some attention was dedicated to the cases of quantum mechanics (QM) based on nonlinear wave equations. One of most elegant attempts was presented by S. Weinberg [1] by applying the Hamiltonian formalism to the complex wave functions. Soon however, it was shown, that the scheme when applied to many particle systems, generates instantaneous messages between distant components in the measurements of Einstein-Podolsky-Rosen (EPR) type (N.Gisin [2], M.Czachor [3]). It was henceforth concluded that the nonlinear QM contradicts the causality. The conclusion has been amended by J.Polchinski [4], who has argued that the superluminal effects are just a special fault of Weinberg's formalism but can be avoided in a wider class of nonlinear theories. Since that time, the idea seems accepted (see, e.g. [5]) without any fundamental critiques. Below, I show that the argument of Polchinski fails: neither the difficulty is specific to the Weinberg's scheme, nor the recipe offered in [4] permits to obtain new types of causal but nonlinear QM.

As it seems, the atypical variants of QM may break with the orthodox scheme in several ways, e.g.: (I) They can modify the manifold of pure states; (II) they can adopt the orthodox (linear) space of pure states but assume the existence of nonlinear evolution

operations; (III) they can adopt the orthodox manifold of pure states but modify the class of the *functional observables* (with or without introducing the nonlinear evolution).

Since the criterion of Polchinski refers to the observables, we shall discuss (III). Consider a pair of hypothetical quantum systems A and B with the pure states described by the unit spheres S_A , S_B in two Hilbert spaces H_A and H_B . Following [2, 4], we neglect the motion of both objects; so H_A and H_B represent the internal degrees. We then adopt the tensor product space $H_A \otimes H_B$ to describe the entangled system (all vectors in $H_A \otimes H_B$ represent the admissible states, the simple products $a \otimes b = |a| > |b| > \text{mean no correlation}$). We also take for granted that all traditional measurements, represented by the orthogonal projectors in H_A , H_B can be performed on the single components of the entangled pair. (We have adopted some essential elements of the orthodox structure, to share the partition point with [2, 3, 1, 4]). In addition, we assume that one of the systems, e.g. B, is atypical in the sense (III), permitting to measure at least one observable $f: S_B \longrightarrow \Re$ which might not be a quadratic form on S_B .

Following EPR, let us now imagine a source which produces a sequence of identical, entangled states:

$$\Psi = \alpha_1 | A_1 > | b_1 > + \dots + \alpha_n | A_n > | b_n >$$
 (1)

bombarding (with a fixed frequency) two distant observers, 'Alice' and 'Bob'. Alice obtains A-objects; she tries to affect the entangled system at her end by performing measurements on |A| states; Bob will try to use f to read the Alice doing. Since the Alice measurements reduce |A>-states to orthogonal systems, we loose little by assuming that $|A_i>(i=$ 1, ... n) are orthonormal. We don't assume the same about $|b_i>$'s, but only that $|b_i>$ = 1 (i = 1,...n) and $|\alpha_1|^2 + ... + |\alpha_n|^2 = 1$. Thus, all simple products in (1) are mutually orthogonal in $H_A \otimes H_B$ and $\langle \Psi | \Psi \rangle = 1$. Suppose that Alice measures an observable A with (nondegenerate) eigenvalues λ_i on eigenstates $|A_i| > (i = 1, ...n)$. If she obtains λ_i , Bob 'receives' the pure state $|b_i| > (\text{note}, \text{ that the justification does not necessarily})$ involves the v. Neumann projection postulate applied in $H_A \otimes H_B$; as long as our theory includes the traditional measurements on each subsystem, the strict statistical correlation on both ends gives as credible argument; compare the 'teleportation' [6]). Thus, if Alice performs a sequence of A measurements on her side, Bob will receive a random sequence $b = |b_1\rangle, |b_2\rangle, |b_3\rangle, ...,$ each $|b_i\rangle$ repeating itself with the frequency $p_i = |\alpha_i|^2$. Suppose now, Alice switched to a new apparatus A' with new (orthonormal) eigenstates $|A'_1>$ $,...,|A'_n>(|A'_i>$ and $|A_i>$ spanning the same subspace of H_A). The entangled state (1) admits an alternative expression:

$$\Psi = \alpha_1' | A_1' > | b_1' > + \dots + \alpha_n' | A_n' > | b_n' >$$
 (2)

where the unit vectors $|b'_i\rangle$ and the coefficients α'_i can be easily calculated. Now, if Alice measures A', Bob receives a new sequence of states $b'=|b'_1\rangle, |b'_2\rangle, ...$ appearing with the new frequencies $p'_i=|\alpha'_i|^2$. Since the single states are not recognizable, the entire sequencies b and b' must be the 'letters' of Alice alphabet. Can Bob read them? To distinguish b and b' he has the conventional observables (of no use!), but he can apply also the observable f. By measuring f on b, he finds the statistical average:

$$f[b] = p_1 f(|b_1\rangle) + \dots + p_n f(|b_n\rangle), \tag{3}$$

while for b' he obtains:

$$f[b'] = p'_1 f(|b'_1\rangle) + \dots + p'_n f(|b'_n\rangle). \tag{4}$$

These averages are also considered by Weinberg, though questioned by Polchinski. Yet, there is some quid pro quo in [4], almost like in Esher's drawings [7]. Indeed, if one does not insist on the orthodox scheme of 'operator observables' [8], then the observables are just c-number functions on states, representing the statistical averages [9, 10, 11, 1]. It means that some universal facts concerning the statistical ensembles must be valid. If an ignorant observer measures f for a sequence of randomly received states, without knowing which is which, he must unavoidably find the statistical averages (3-4). Thus, (3-4) have a universal validity. (At least, nothing can stop Bob from making precisely this statistics at his end!). The concept of a "density matrix", meanwhile, is particular; in fact, it turns insufficient to describe the mixed states in nonlinear theories [9, 10, 11]. What Polchinski assumes is that the density matrices of the B-subsystem, still contain enough information to determine the values (3-4) for the observable f on the b-sequencies. If one adopts the idea, the rest of the story develops in H_B . If b and b' are 'generated' by Alice (by measuring A and A'), then the simple calculation shows that their 'density matrices' coincide:

$$\rho = \sum |\alpha_i|^2 |b_i| < b_i| = \sum |\alpha_i'|^2 |b_j'| > < b_k'| = \rho'$$
(5)

The criterion [4] then says f[b] = f[b']; so f does not distinguish Alice letters. One might hope that by choosing arbitrary $f(\rho)$ one can arrive at distinct no-signal theories, but this turns out an illusion. The point is that an observable (statistical average) cannot be postulated without caring for the consistency conditions, which interrelate its values on the mixture with the values on the mixture components. As a consequence, if (3) and (4) coincide for any two sequencies generated by Alice (yielding a well defined function of ρ), then f can be only a quadratic form on S_B . To illustrate this, take dim $H_B = 2$. The convex set of all density matrices in H_B can be represented as the unit ball R_1 in \Re^3 . The ball surface S^2 , (i.e., the projective unit sphere in H_B) collects the simple

density matrices of the form |b> < b| (rays in H_B). We stick to the assumption that they represent the pure states of the B-subsystem. The antipodal points of S^2 stand for orthogonal rays. The "density matrices" in H_B are arbitrary points $x \in R_1$ (Fig.1); the convex linear combinations $p_1x_1 + p_2x_2$ ($p_1, p_2 \ge 0$, $p_1 + p_2 = 1$) for $x_1, x_2 \in R_1$ define the natural geometry of R_1 [9]. We adopt the idea [4] that they contain some (at least partial) information about the physical mixtures and that p_1, p_2 are the mixing probabilities. Consider now two pairs of points (pure states) x_1, x_2 and x'_1, x'_2 on S^2 . Following [4], we assume that if the straight line intervals x_1x_2 and $x'_1x'_2$ intersect at a point $x = p_1x_1 + p_2x_2 = p'_1x'_1 + p'_2x'_2 \in R_1$, the values of f on both mixtures must coincide:

$$p_1 f(x_1) + p_2 f(x_2) = p_1' f(x_1') + p_2' f(x_2').$$
(6)

so that (6) becomes a well defined function $\Phi(x)$ of $x = p_1x_1 + p_2x_2 \in R_1$:

$$p_1 f(x_1) + p_2 f(x_2) = \Phi(p_1 x_1 + p_2 x_2). \tag{7}$$

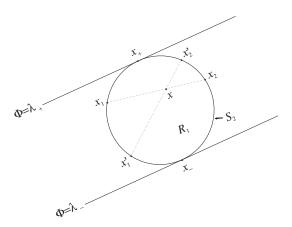


Figure 1: Due to the natural geometry of the density matrices in dim $H_B = 2$, the 'no-signal condition' of Polchinski can be satisfied only by the affine functionals in \Re^3 , corresponding to the quadratic observables $f: S_B \longrightarrow \Re$.

By physical arguments, Φ should be continuous. Putting $p_1 = 1$, $p_2 = 0$ or $p_1 = 0$, $p_2 = 1$, one gets $f(x_1) = \Phi(x_1)$ and $f(x_2) = \Phi(x_2)$, which converts (7) into:

$$\Phi(p_1x_1 + p_2x_2) = p_1\Phi(x_1) + p_2\Phi(x_2), \tag{8}$$

i.e., Φ is linear with respect to the convex combination in R_1 . Since R_1 spans \Re^3 , it is the matter of simple extension to consider Φ linear on \Re^3 with respect to the affine linear combination $p_1x_1 + p_2x_2$, $(p_1, p_2 \in \Re, p_1 + p_2 = 1)$. If $\Phi \neq \text{const}$ in \Re^3 , then the equations $\Phi = \text{const}$ determine a congruence of closed, parallel planes in \Re^3 . Two of them are tangent to $S^2 = \partial R_1$ in two antipodal points $x_{\pm} = |b_{\pm}| > < b_{\pm}|$ where Φ accepts its

maximal and minimal values λ_{\pm} (on R_1). Exactly the same properties has the functional $\lambda_{+} < b_{+}|x|b_{+} > +\lambda_{-} < b_{-}|x|b_{-} >$. Thus: $\Phi(x) = \lambda_{+} < b_{+}|x|b_{+} > +\lambda_{-} < b_{-}|x|b_{-} >$. In particular, for $x = |\psi| > \psi$ ($\in S^2$), $\Phi(x) = \lambda_{+}|\psi| > |\psi| > |\psi| > |\psi| > |\psi| > |\psi|$ is just a quadratic form of the pure states ψ . If Φ is constant in \Re^3 , the same holds with $\lambda_{+} = \lambda_{-}$.

Paradoxically, the proof is even simpler if dim $H_B \geq 3$. Let us recall that all quantum measurements can be reduced to elementary 'counting experiments' (carried out by unsophisticated counters which can either detect or overlook the particle). If f is a 'counting observable', then $0 \leq f[b] \leq 1$. Suppose, f satisfies the condition of Polchinski [4]. Let $X \subset H_B$ be a subspace (dim X = n), P_X the corresponding projector and $|b_1 >, ..., |b_n >$ any orthonormal basis in X; then the sum $f[b] = \left(\frac{1}{n}\right) [f(|b_1 >) + ... + f(|b_n >)]$ does not depend on the basis, but only on the entire subspace X. The same concerns the 'renormalized' sum:

$$f(|b_1\rangle) + \dots + f(|b_n\rangle) = nf[b] = \mu(X)$$
 (9)

which therefore defines a non-negative measure μ on the subspaces $X \subset H_B$. By taking two subsequencies $|b_1>,...,|b_r>$ and $|b_{r+1}>,...,|b_n>$, and the corresponding two orthogonal subspaces $Y, Z \subset H_B, Y+Z=X, X\perp Y$, we see from (9) that $\mu(Y)+\mu(Z)=\mu(X)$, i.e., μ is a positively defined, orthoadditive measure on the subspaces $X\subset H_B$. Since dim $H_B\geq 3$, Gleason theorem [12] implies the existence of a non-negative operator $F:H_b\to H_b$, such that for any $X\subset H_b:\mu(X)=Tr(FP_X)$, where P_X are the orthogonal projectors associated with the subspaces $X\subset H_B$. In particular, if X is a 1-dim subspace spanned by the unit vector $|\psi>$, and $P_X=|\psi><\psi|$, then $f(|\psi>)=\mu(X)=Tr(F|\psi><\psi|)=<\psi|F|\psi>$; i.e., f is a quadratic form on S_B . (Notice, that we have slightly strengthened the original Gisin argument [2], by limiting the Polchinski condition for f to the orthogonal b-sequencies). Since any observable is a linear combination of 'counting observables', we have shown that any observable satisfying the Polchinski criterion must be quadratic on S_B . The non-quadratic observables protected against the superluminal effects are an illusion (just give me one non-quadratic form on S_B , representing a statistical average, and nothing can stop me from using (3-4) to read Alice messages!).

We conclude that the superluminal effects are not a specific difficulty of Weinberg's approach, but a generic phenomenon in nonlinear theories which have absorbed too ample fragments of the orthodox scheme. A way out, perhaps, could be a consistent deformation of the pure and mixed states, as well as the functional observables. This is, however, a different story which still waits to be written.

References

- [1] S. Weinberg, Ann. Phys. (N.Y.) 194, 336 (1989)
- [2] N. Gisin, Helv. Phys. Acta **62**,363 (1989); Phys. Lett. **A 143**, 1 (1990)
- [3] M. Czachor, Found. Phys. Lett. 4, 351 (1991)
- [4] J. Polchinski *Phys.Rev.Lett.* **66**, 397 (1991)
- [5] M. Czachor, *Phys.Rev.* A **57**, 4122 (1998)
- [6] C.H. Bennett, G.Brassard, C.Cre'peau, R.Jozsa, A.Peres, and W.Wooters, Phys.Rev.Lett. 70, 1895 (1993)
- [7] M. C. Esher, Belvedere in Grafiek en Tekeningen, ed. B. Taschen Verlag, Köln (1994)
- [8] P.A.M. Dirac, "Principles of Quantum Mechanics", Clarendon, Oxford (1958)
- [9] B. Mielnik, Commun. Math. Phys. 15, 1 (1969); 31, 221 (1974)
- [10] R. Haag and U. Bannier, Commun. Math. Phys. **60**, 1 (1978)
- [11] T.W. Kibble and S. Randjbar Daemi, J. Phys. A 13, 141 (1980)
- [12] A.M. Gleason, J.Math and Mech. 6, 885 (1957)